VIII. *Aſtronomical Obſervations. In two Letters from M.* Francis de Zach, *Profeſſor of Mathematics, and Member of the Royal Academies of Sciences at* Marſeilles, Dijon, *and* Lyons, *to Mr.* Tiberius Cavallo, *F. R. S.*

Read December 23, 1784.

SIR, Lyons, April 4, 1783.

I SEND you the account of the obſervations on the eclipſe of the moon, which I have made together with the rev. Father LE FEVRE, Aſtronomer at Lyons, in the Obſervatory called *au grand Collège*; to which I ſhall add the obſervations of the vernal equinox; ſome obſervations on Jupiter's ſatellites, made at Marſeilles by M. SAINT JAQUES DE SYLVABELLE; and, laſtly, a new ſolution of a problem that occurs in computing the orbits of comets. If you think that theſe obſervations do in any way deſerve the notice of the Royal Society, I ſhall be very glad you would communicate them. In order to aſcertain the going of the pendulum clock, I took ſeveral correſponding altitudes of the ſun, which you will find in the following table. On the day of the eclipſe the ſky was very ſerene, nothing could be finer, and it continued ſo during the obſervation. I determined to uſe an achromatic teleſcope of 3½ feet length, that ſhews objects in their natural poſition, becauſe the diluted and uncertain termination of the true ſhadow of the earth appears more perfectly defined by ſmall than by

large

large teleſcopes, which magnify too much, and give too great a tranſit between the penumbra and the true dark ſhadow. On that account ſome celebrated aſtronomers adviſe to uſe for the eclipſes of the moon no greater teleſcopes than of four or five feet length. It was remarked at Paris, that in an eclipſe of the moon, obſerved through a teleſcope of DOLLOND, the focus of its object lens being 30 inches, and likewiſe through a teleſcope of five feet length; the eclipſe appeared to begin 4′ 7″ ſooner, and to end 4′ 7″ later, through the ſmall than through the long teleſcope; the like has been remarked by ſeveral others, and it has been alſo obſerved by myſelf. As to my obſervations I am tolerably ſatisfied with them, as they do not differ materially from thoſe of Father LE FEVRE, though it is known that in eclipſes of the moon no greater exactneſs than that of a minute can be obtained. The moon's ſpots were carefully obſerved; for it is known, that the mean of the obſervations of the moon's ſpots is ſufficient to aſcertain the longitude of a place to 4″ or 5″ nearly. M. DE LA LANDE comparing the obſervations of the moon's ſpots in an eclipſe, made the 22d of November, 1760, in Vienna, by the Imperial Aſtronomer Abbé HELL, with thoſe made at the ſame time in Paris by M. MESSIER, finds the difference of meridians to be 56′ 13″, which agrees very exactly with that aſcertained by other means.

Correſpondent altitudes of the Sun taken with a quadrant of three-feet radius, in order to adjuſt the pendulum clock to apparent time.

18th March, 1783.	Sun's altit.	Iſt obſervation.	Sun's altit.	IId obſervation.	Sun's altit.	IIId obſervation.	Sun's altit.	IVth obſervation.	Sun's altit.	Vth obſervation.
	° '	h. ' ''	° '	h. ' ''	° '	h. ' ''	° '	h. ' ''	°	h. ' ''
The ſun's upper limb at the horizontal wire, eaſtern ſide	25 40	8 25 2	26 10	8 28 25	26 40	8 31 43	27 20	8 36 16	28	8 40 53
Sun's upper limb at the ſame altitude, weſtern ſide -		15 12 24		15 9 1		15 5 39		15 1 10		14 56 29
Dividing the ſum by 2		23 37 26		23 37 26		23 37 22		23 37 26		23 37 22
Sun's center on the meridian as marked by the clock		11 48 43		11 48 43		11 48 41		11 48 43		11 48 41
Equation of the day		12 8 15		12 8 15		12 8 15		12 8 15		12 8 15
Clock ſlower than equated ſolar time		0 19 32		0 19 32		0 19 34		0 19 32		0 19 34

The mean of which I put 19′ 32″

19th March, 1783.	Sun's Altit.	Iſt obſervation.	Sun's Alt.	IId obſervation.
	° '	h. ' "	°	h. ' "
The ſun's upper limb at the horizontal wire of the firſt teleſcope on the eaſtern ſide - - -	31 30	9 1 56	33	9 13 9
Sun's upper limb at the ſame altitude on the weſtern ſide of the meridian -		14 31 47		14 20 33
Dividing the ſum by 2 - - -		23 33 43		23 33 42
Sun's center on the meridian as marked by the pendulum clock - -		11 46 51½		11 46 51
Equation of the day		12 7 57		12 7 57
Clock ſlower than equated ſolar time		0 21 6		0 21 6

		'	"
Clock ſlower than equated ſolar time	19th March	21	6
—— —— ——	18th March	19	32
Retarding of the clock upon 23 h. 58' 8" -		1	34

I obſerved too the mid-day at the great gnomon of the obſervatory, and found at the ſame time the meridian line erroneous by 19", as you will find in the following tables.

		h. ' "		h. ' "		h. ' "
When the center of the ſun's image was on the meridian the time pointed by the clock was	the 17th Mar.	11 50 50	the 18th Mar.	11 48 56	the 19th Mar.	11 47 3
Equations of thoſe days -		12 8 33		12 8 15		12 7 57
Retarding on equated ſolar time		17 43		19 19	the	20 54
Retarding the 17th - -				17 43	18th	19 19
Retarding of the clock during thoſe 24 hours				1 36		1 35

I fixed therefore the retarding of the clock 1' 35".

		h.	′	″		h.	′	″
True mid-day concluded by the ſun's correſpondent altitudes as the clock marked	the 18th Mar.	11	48	42	the 19th Mar.	11	46	51
Equation of the mid-day			—	18			—	18
Retarding of the clock at the rate of 1′ 35″ per 24 hours			+	13			+	11
True mid-day the pendulum clock ſhould have marked		11	48	37		11	46	44
Mid-day concluded at the gnomon of the obſervatory		11	48	56		11	47	3
Difference, the error of the meridian line or gnomon				19				19

From thence I concluded,

	h.	′	″		h.	′	″
Mid-day at true ſolar time	11	59	60	Mid-day at equated ſolar time	12	8	15
Mid-day the clock ſhould have marked on the 18th	11	48	37	Mid-day the clock ſhould have marked on the 18th	11	48	37
Retarding upon true ſolar time		11	23	Retarding upon equated ſolar time		19	38

Obſervations of the moon's eclipſe the 18th March, 1783.

My obſervations with an achromatic teleſcope of 3½ feet length.	Time marked by the clock.	True or apparent time.
IMMERSIONS.		
The beginning of the eclipſe very doubtful.	h. ′ ″	h. ′ ″
Shadow touches Grimaldi	7 41 45	7 53 39
Grimaldi all in the ſhadow	7 42 54	7 54 48
Shadow touches Mare Humorum	7 49 32	8 1 27
———— Copernicus	8 0 21	8 12 16
Copernicus all in the ſhadow	8 2 29	8 14 24
Tycho touches the ſhadow	8 6 18	8 18 13
Eudoxus ————	8 12 19	8 24 15
——— all in the ſhadow	8 13 43	8 25 39
Mare Serenitatis touches the ſhadow	8 16 7	8 28 3
———— the ſhadow in the middle	8 21 31	8 33 27
———— all in the ſhadow	8 25 54	8 37 50
Proclus touches the ſhadow	8 32 21	8 44 18
Mare Criſium touches the ſhadow	8 33 29	8 45 26
———— ſhadow in the middle	8 35 36	8 47 33
———— all in the ſhadow	8 36 56	8 48 53
Total immerſion	8 38 57	8 50 55
EMERSIONS.		
Beginning of the emerſion	10 19 57	10 32 2
Grimaldi emerging	10 23 33	10 35 38
——— all out of the ſhadow	10 24 9	10 36 14
Mare Humorum emerging	10 29 34	10 41 39
Total emerſion of Mare Humorum	10 35 37	10 47 43
Copernicus all out of the ſhadow	10 43 6	10 55 12
Mare Serenitatis all emerged	10 57 32	11 9 39
Mare Criſium all emerged	11 15 44	11 27 51
End of the eclipſe	11 20 10	11 32 18
Total duration		3 39 0

Father LE FEVRE's obſervations with a reflector 55 inches focal length, magnifying 300 times.

	Time by the clock.			Apparent time.		
IMMERSIONS.	h.	′	″	h.	′	″
Grimaldi touches the ſhadow	7	41	43	7	53	37
Kepler touches the ſhadow	7	52	2	8	3	57
——— all in the ſhadow	7	53	24	8	5	19
Copernicus touches the ſhadow	8	0	22	8	12	17
——— all in the ſhadow	8	2	26	8	14	21
Mare Serenitatis touches the ſhadow	8	16	7	8	28	3
——— all in the ſhadow	8	26	2	8	37	58
Mare Criſium touches the ſhadow	8	33	28	8	45	25
——— all in the ſhadow	8	36	56	8	48	53
Total immerſion	8	38	54	8	50	52
EMERSIONS.						
Beginning of the emerſion	10	19	42	10	31	47
Grimaldi emerged	10	23	24	10	35	29
Kepler all out of the ſhadow	10	35	43	10	47	49
Copernicus all out	10	43	4	10	55	10
Mare Serenitatis all out	10	57	19	11	9	26
——— Criſium all out	11	15	50	11	27	57
End of the eclipſe	11	20	22	11	32	30
Total duration				3	39	20

The obſervation of the vernal equinox was made at the gnomon. The height of this gnomon, taken from the center of the hole by which the beams of the ſun come in, is 1878 lines of a French inch; the diſtance from the bottom of the gnomon to the equinoctial point is 1928; the diſtance from the upper limb of the ſun's image to the equinoctial point was found 16,7; the diſtance from the under limb 23,4; the diameter of the hole $= 6$; therefore the diſtance from the bottom to the upper limb $1928 - 16{,}7 + 3 = 1914{,}3$, to the under limb $1928 + 23{,}4 - 3 = 1948{,}4$; which gives the time the equinox happened the 20th of March, 5 h. 56′ 52″ P.M.

Obſervations of Jupiter's ſatellites at Marſeilles.

1782		Apparent time.			Obſervation.
		h.	′	″	
April 3	Immerſion of the Iſt ſatellite at	2	22	56	good
May 19	Imm. Iſt ſat.	2	48	12	good
June 7	Imm. IVth ſat. was not total, but its light diminiſhed ſenſibly without ever diſappearing; the ſky was ſerene, and Jupiter had ſix belts very diſtinctly.				
20	Emerſion of the Iſt ſatellite	1	29	46	good
July 5	Em. Iſt ſat.	11	43	59	good
13	Em. IId ſat.	9	17	28	good
20	Em. IIId ſat.	9	27	13	good
20	Em. IId ſat.	11	51	59	good
21	Em. Iſt ſat.	10	1	11	good
27	Imm. IIId ſat.	10	40	33	good
Aug. 6	Em. Iſt ſat.	8	21	20	good
13	Em. Iſt ſat.	10	18	49	good
14	Em. IId ſat.	8	55	39	good
Sept. 1	Em. IIId ſat.	9	40	44	doubtful
14	Em. Iſt ſat.	8	6	48	good

IT is known, that the indirect method to calculate the orbits of comets in a conic ſection, by means of three obſervations given, is rendered more eaſy and expeditious if there is a poſſibility of drawing a graphical figure that repreſents nearly the orbit under conſideration, by means of which the calculation is directed, and the required elements of the comet's path may be rigorouſly determined. To draw the orbit of a comet that moves in a parabola or ellipſis, the problem is reduced to find the poſition of the axis and the perihelial diſtance; this poſition of the axis will be determined as ſoon as the angle is known, that the axis forms with another line, whoſe poſition is given; this line may be an ordinate to a given point of the curve, or a tangent, or a radius vector, &c. The latter is to be employed in preference, becauſe the perihelial diſtance being a conſtant quantity, the angle of poſition then becomes the true anomaly of the comet; but as the data of this problem are only geocentric longitudes and latitudes of the comet, deduced from the immediate obſervations of right aſcenſion and declination, the heliocentric longitudes and latitudes muſt firſt be calculated; but as thoſe data are not ſufficient, what is not given muſt be arbitrarily ſuppoſed, *viz.* the ſhortened diſtances (*diſtantias curtatas*). This ſuppoſition is changed and altered until the calculation will agree with the three obſervations, then the difference between two longitudes is the angle comprehended between the two ſhortened diſtances in the plane of the ecliptic; the whole reduced to the plane of the comet's orbit by means of the heliocentric latitude, gives the difference between the anomalies comprehended by two radius vectors, the problem then is reduced: two radius vectors being given, with the angle comprehended, to find the two true anomalies, the perihelial diſtance, and the time the comet puts in running its anomalies.

Let therefore ♈ ♋ ♎ ♑ repreſent the ecliptic at an infinite diſtance; QPR the apparent elliptical or parabolical path of a comet; S the ſun's center; P the comet's perihelion; T the place of the earth when the comet was firſt obſerved in C; I the earth's place when the comet was obſerved in K; ST $= d$, SI $= \delta$, the diſtances from the earth to the ſun at the firſt and ſecond obſervation known by aſtronomical tables; let Cm and Kn be two perpendiculars to the plane of the ecliptic, it will be S$m = u$, S$n = v$ the two ſhortened diſtances.

The obſerved geocentric longitude of the comet in T $= a =$ arc ♈ ♑ G;
the obſerved geocentric longitude of the comet in I $= \alpha =$ arc ♈ ♑ H;
the geocentric longitude of the ſun by tables in T $= b =$ arc ♈ ♑ ♎ A;
the geocentric longitude of the ſun by tables in I $= \beta =$ arc ♈ ♑ ♎ B.

Now for the firſt obſervation the angle of elongation is $b - a$; for the angle ATG = arc AG = ♈♑♎A − ♈♑G = long. ☉ − long. comet. $= b - a$;

the angle of the annual parallax $SmT = \frac{\text{ſin.}(b-a)d}{u} = e$;

the angle of commutation $mST = \overset{0^\circ}{180^\circ} - e + (b - a) = f$;

from whence the heliocentric longitude of the comet $= b - 180^\circ + f = g$.

The ſame at the ſecond obſervation in I.

Angle of elongation $= \beta - \alpha$;

Angle of annual parallax $\varepsilon = \frac{\text{ſin.}\,(\beta-\alpha)\,\delta}{v}$;

Angle of commutation $\phi = \overset{0^\circ}{180^\circ} - \varepsilon + (\beta - \alpha)$;

heliocentric longitude of the comet in $I = \beta - 180^\circ + \phi = \gamma$;

putting now the heliocentric latitude ſeen from $S = k$;

the geocentric latitude ſeen from $T = l$;

the heliocentric latitude will be $\frac{\text{ſin.}\,f\,.\,\text{tang.}\,l}{\text{ſin.}\,(b-a)} = \text{tang.}\,k$;

the ſame with Kn it will be $\frac{\text{ſin.}\,\phi\,.\,\text{tang.}\,\lambda}{\text{ſin.}\,(\beta-\alpha)} = \text{tang.}\,\varkappa$ heliocentric latitude in K.

Having thus determined the heliocentric latitudes of two obſervations, the radius vectors will eaſily be found in the ſuppoſition made for the ſhortened diſtances, for they are in the ſame ratio to the radius vectors as the coſine of the heliocentric latitudes are to the radius = 1; therefore the radius vector m of the firſt obſervation will be $= \frac{u}{\text{coſ.}\,k}$ and the radius vector of the ſecond obſervation $\mu = \frac{v}{\text{coſ.}\,\varkappa}$.

Taking now the difference between the found heliocentric longitudes, we get the heliocentric motion of the comet upon the ecliptic between two ſhortened diſtances, which is to be reduced upon the comet's orbit, this heliocentric motion is therefore $\gamma - g = m$. Now to reduce this motion we have, firſt, ſinus totus = 1 is to coſine m :: as cotangent k is to the tangent of an angle which I put = n, and $90^\circ - k \mp n$ will give an angle which I put = q. Laſtly, the analogy coſ. n : coſ. q :: ſin. k : will give the coſine of an angle ψ, which is the required motion upon the orbit, or the angle comprehended between the two radius vectors m and μ. Let therefore ECPMND be the apparent parabolic path of a comet; S the ſun's center; M and N two places of the comet, the angle MSN equal to its motion in longitude, or the comprehended angle ψ; P the perihelion; it is required to find the two anomalies PM, PN, that is, PSM and PSN, the perihelial diſtance SP, and the time the comet employed to come from its perihelion P to M and N.

Reſolution.

$SM = m$
$SN = \mu$
$MSN = \psi$
$NSB = x$
$MSB = (\psi \pm x)$
Parameter $= p$

In the right-angled triangle SMR and SNV we have $MR = OS = m\,\text{ſin.}\,(\psi \pm x)$ $NV = QS = \mu\,\text{ſin.}\,x$; therefore $OP = \frac{1}{4}p - m\,(\text{ſin.}\,\psi \pm x)$ and $PQ = \frac{1}{4}p \mp \mu\,\text{ſin.}\,x$; but by the nature of the parabola we have SM = AP + PO and SN = AP + PQ; that is

$$m = \tfrac{1}{2}p - m\,(\text{ſin.}\,\psi \pm x) \qquad \mu = \tfrac{1}{2}p \mp \mu\,\text{ſin.}\,x$$

$$m + m\,(\text{ſin.}\,\psi \pm x) = \tfrac{1}{2}p \qquad \mu \pm \mu\,\text{ſin.}\,x = \tfrac{1}{2}p$$

$$m\,(1 + \text{ſin.}\,\psi \pm x) = \tfrac{1}{2}p \qquad \mu\,(1 \mp \text{ſin.}\,x) = \tfrac{1}{2}p$$

and $1 + \text{ſin.}\,(\psi \pm x) = \frac{p}{2m}$ $\qquad 1 \pm \text{ſin.}\,x = \frac{p}{2\mu}$; by putting into a ſum $1 + \text{ſin.}\,(\psi \pm x) + 1 \pm \text{ſin.}\,x = \frac{p}{2m} + \frac{p}{2\mu}$; reduction made $2 \pm \text{ſin.}\,x + \text{ſin.}\,(\psi \pm x) = \left(\frac{m+\mu}{2m\mu}\right)p$; but by trigonometrical formulæ we have $\text{ſin.}\,(\psi \pm x) = \text{ſin.}\,\psi\,\text{coſ.}\,x \pm \text{ſin.}\,x\,\text{coſ.}\,\psi$. Subſtituting this expreſſion in its place we obtain, $2 \pm \text{ſin.}\,x + \text{ſin.}\,\psi\,\text{coſ.}\,x \pm \text{ſin.}\,x\,\text{coſ.}\,\psi = \left(\frac{m+\mu}{2m\mu}\right)p$. By the ſame formulæ we have $\text{coſ.}^2\,x = 1 - \text{ſin.}^2\,x$ and $\text{coſ.}\,x = \sqrt{1 - \text{ſin.}^2\,x}$.

Sub-

Subſtituting it comes out, $2 \pm \text{ſin.}\, x + \text{ſin.}\,\psi\sqrt{1 - \text{ſin.}^2 x} \pm \text{ſin.}\, x\, \text{coſ.}\,\psi = \left(\frac{m+\mu}{2m\mu}\right) p$ and $\text{ſin.}\,\psi\sqrt{1-\text{ſin.}^2 x} = \left(\frac{m+\mu}{2m\mu}\right)p - 2 \mp \text{ſin.}\,x \mp \text{ſin.}\,x\,\text{coſ.}\,\psi$. By caſting away the ſign of the ſquare root, and reducing to o,

$$\left(\frac{m+\mu}{4m^2\mu^2}\right)^2 p^2 + 4 + \text{ſin.}^2 x + \text{ſin.}^2 x\,\text{coſ.}^2\psi - 2\left(\frac{m+\mu}{m\mu}\right)p \mp \left(\frac{m+\mu}{m\mu}\right)p\,.\,\text{ſin.}\,x \mp \left(\frac{m+\mu}{m\mu}\right)p\,.\,(\text{ſin.}\,x\,\text{coſ.}\psi) \pm 4\,\text{ſin.}\,x\,\text{coſ.}\,\psi + 2\,\text{ſin.}^2 x\,\text{coſ.}\,\psi - \text{ſin.}^2\psi + \text{ſin.}^2 x\,\text{ſin.}^2\psi = 0;$$

diſentangling the equation,

$$\frac{m^2p^2 + 2m\mu p^2 + \mu^2 p^2}{4m^2\mu^2} + 4 + \text{ſin.}^2 x + \text{ſin.}^2 x\,\text{coſ.}^2\psi - \frac{2mp - 2\mu p}{m\mu} \mp \frac{mp\,\text{ſin.}\,x \mp \mu p\,\text{ſin.}\,x}{m\mu} \mp \frac{mp\,\text{ſin.}\,x\,\text{coſ.}\,\psi \mp \mu p\,\text{ſin.}\,x\,\text{coſ.}\,\psi}{m\mu} \mp 4\,\text{ſin.}\,x \pm 4\,\text{ſin.}\,x\,\text{coſ.}\,\psi + 2\,\text{ſin.}^2 x\,\text{coſ.}\,\psi - \text{ſin.}^2\psi + \text{ſin.}^2 x\,\text{ſin.}^2\psi = 0.$$

But p being equal to $2\mu \pm 2\mu\,\text{ſin.}\,x$, ſubſtituting this value, we have

$$\frac{4m^2\mu^2 \pm 8m^2\mu^2\,\text{ſin.}\,x + 4m^2\mu^2\,\text{ſin.}^2 x + 8m\mu^3 + 8m\mu^3\,\text{ſin.}^2 x \pm 16m\mu^3\,\text{ſin.}\,x + 4\mu^4 + 4\mu^4\,\text{ſin.}^2 x \pm 8\mu^4\,\text{ſin.}\,x}{4m^2\mu^2} + 4 + \text{ſin.}^2 x +$$

$$+ \text{ſin.}^2 x\,\text{coſ.}^2\psi - \frac{4m\mu \mp 4m\mu\,\text{ſin.}\,x - 4\mu^2 \mp 4\mu^2\,\text{ſin.}\,x \mp 2m\mu\,\text{ſin.}\,x - 2m\mu\,\text{ſin.}^2 x \mp 2\mu^2\,\text{ſin.}\,x - 2\mu^2\,\text{ſin.}^2 x \mp}{m\mu}$$

$$\frac{\mp 2m\mu\,\text{ſin.}\,x\,\text{coſ.}\,\psi - 2m\mu\,\text{ſin.}^2 x\,\text{coſ.}\,\psi \mp 2\mu^2\,\text{ſin.}\,x\,\text{coſ.}\,\psi - 2\mu^2\,\text{ſin.}^2 x\,\text{coſ.}\,\psi}{m\mu} \pm 4\,\text{ſin.}\,x \pm 4\,\text{ſin.}\,x\,\text{coſ.}\,\psi + 2\,\text{ſin.}^2 x\,\text{coſ.}\,\psi - \text{ſin.}^2\psi +$$

$$+ \text{ſin.}^2 x\,\text{ſin.}^2\psi = 0.$$

Dividing the numerator and denominator of the firſt fraction by $4\mu^2$, and of the ſecond by μ, and putting the whole to the common denominator m^2 it comes

$$\frac{m^2 \pm 2m^2\,\text{ſin.}\,x + m^2\,\text{ſin.}^2 x + 2m\mu + 2m\mu\,\text{ſin.}^2 x \pm 4m\mu\,\text{ſin.}\,x + \mu^2 + \mu^2\,\text{ſin.}^2 x \pm 2\mu^2\,\text{ſin.}\,x + 4m^2 + m^2\,\text{ſin.}^2 x\,\text{coſ.}^2\psi + m^2\,\text{ſin.}^2 x -}{m^2}$$

$- 4m^2$

$$\frac{-4m^2 \mp 4m^2\,\text{ſin.}\,x - 4m\mu \mp 4m\mu\,\text{ſin.}\,x \mp 2m^2\,\text{ſin.}\,x - 2m^2\,\text{ſin.}^2 x \mp 2m\mu\,\text{ſin.}\,x - 2m\mu\,\text{ſin.}^2 x \mp 2m^2\,\text{ſin.}\,x\,\text{coſ.}\,\psi - 2m^2\,\text{ſin.}^2 x\,\text{coſ.}\,\psi \mp}{m^2}$$

$$\frac{\mp 2m\mu\,\text{ſin.}\,x\,\text{coſ.}\,\psi - 2m\mu\,\text{ſin.}^2 x\,\text{coſ.}\,\psi \pm 4m^2\,\text{ſin.}\,x \pm 4m^2\,\text{ſin.}\,x\,\text{coſ.}\,\psi + 2m^2\,\text{ſin.}^2 x\,\text{coſ.}\,\psi - m^2\,\text{ſin.}^2\,\psi + m^2\,\text{ſin.}^2 x\,\text{ſin.}^2\,\psi}{m^2} = 0.$$

Caſting away m^2, and reducing, we have

$$m^2 + \mu^2 + \mu^2\,\text{ſin.}^2 x \pm 2\mu^2\,\text{ſin.}\,x + m^2\,\text{ſin.}^2 x\,\text{coſ.}^2\,\psi - 2m\mu \mp 2m\mu\,\text{ſin.}\,x\,\text{coſ.}\,\psi - 2m\mu\,\text{ſin.}^2 x\,\text{coſ.}\,\psi - m^2\,\text{ſin.}^2\,\psi + m^2\,\text{ſin.}^2 x\,\text{ſin.}^2\,\psi \mp$$
$$\mp 2m\mu\,\text{ſin.}\,z \pm 2m^2\,\text{ſin.}\,x\,\text{coſ.}\,\psi = 0.$$

Putting together ſin.^{2} z and ſin. x

$$(\mu^2 + m^2\,\text{coſ.}^2\,\psi - 2m\mu\,\text{coſ.}\,\psi + m^2\,\text{ſin.}^2\,\psi)\,\text{ſin.}^2 x \pm (2\mu^2 \mp 2m\mu\,\text{coſ.}\,\psi \mp 2m\mu \pm 2m^2\,\text{coſ.}\,\psi)\,\text{ſin.}\,x + m^2 - 2m\mu + \mu^2 - m^2\,\text{ſin.}^2\,\psi = 0.$$

Subſtituting coſ.$^2\,\psi = 1 -$ ſin.$^2\,\psi$ we recover a quadratic equation,

$$(\mu^2 + m^2 - 2m\mu\,\text{coſ.}\,\psi)\,\text{ſin.}^2 x \pm (2\mu^2 \mp 2m\mu\,\text{coſ.}\,\psi \pm 2m^2\,\text{coſ.}\,\psi \mp 2m\mu)\,\text{ſin.}\,x + m^2 - 2m\mu + \mu^2 - m^2\,\text{ſin.}^2\,\psi = 0,$$

and $\text{ſin.}^2 x \pm \frac{(2\mu^2 \mp 2m\mu\,\text{coſ.}\,\psi \pm 2m^2\,\text{coſ.}\,\psi \mp 2m\mu)}{(\mu^2 + m^2 - 2m\mu\,\text{coſ.}\,\psi)}\;\text{ſin.}\,x = \frac{2m\mu - m^2 - \mu^2 + m^2\,\text{ſin}^2\,\psi}{\mu^2 + m^2 - 2m\mu\,\text{coſ.}\,\psi}$.

This equation reſolved gives

$$\text{ſin.}\,x \pm \frac{\mu^2 \mp m\mu\,\text{coſ.}\,\psi \pm m^2\,\text{coſ.}\,\psi \mp m\mu}{\mu^3 + m^2 - 2m\mu\,\text{coſ.}\,\psi} = \sqrt{\frac{2m\mu - m^2 - \mu^2 + m^2\,\text{ſin.}^2\,\psi}{\mu^2 + m^2 - 2m\mu\,\text{coſ.}\,\psi} \pm \frac{(\mu^2 \mp m\mu\,\text{coſ.}\,\psi \pm m^2\,\text{coſ.}\,\psi \mp m\mu)^2}{(\mu^2 + m^2 - 2m\mu\,\text{coſ.}\,\psi)^2}}$$

which gives farther,

$$\text{ſin.}\,x = \frac{\sqrt{(2m\mu - m^2 - \mu^2 + m^2\,\text{ſin.}^2\,\psi)\,.\,(\mu^2 + m^2 - 2m\mu\,\text{coſ.}\,\psi) \pm (\mu^2 \mp m\mu\,\text{coſ.}\,\psi \pm m^2\,\text{coſ.}\,\psi \mp m\mu)^2}}{\mu^2 + m^2 - 2m\mu\,\text{coſ.}\,\psi} \mp \frac{\mu^2 \mp m\mu\,\text{coſ.}\,\psi \pm m^2\,\text{coſ.}\,\psi \mp m\mu}{\mu^2 + m^2 - 2m\mu\,\text{coſ.}\,\psi}.$$

Putting $1 -$ ſin.$^2\,\psi$ inſtead of coſ.$^2\,\psi$, after all reduction made, we recover

$$\text{ſin.}\,x = \frac{\sqrt{2m^3\mu\,\text{ſin.}^2\,\psi - 2m^3\mu\,\text{coſ.}\,\psi\,\text{ſin.}^2\,\psi}}{m^2 + \mu^2 - 2m\mu\,\text{coſ.}\,\psi} \pm \left(\mp \frac{\mu^2 \pm \mu m\,\text{coſ.}\,\psi \mp m^2\,\text{coſ.}\,\psi \pm m\mu}{\mu^2 + m^2 - 2m\mu\,\text{coſ.}\,\psi}\right);$$

that is,

$$\text{ſin.}\,x = (\pm\mu \mp m)\,.\,(m\,\text{coſ.}\,\psi - \mu) \pm \sqrt{2m^3\mu\,\text{ſin.}^2\,\psi\,(1 - \text{coſ.}\,\psi)},$$

which gives at laſt the ſimple expreſſion

$$\text{ſin.}\,x = \frac{(\pm\mu \mp m)\,.\,(m\,\text{coſ.}\,\psi - \mu) \pm m\,\text{ſin.}\,\psi\sqrt{2m\mu\,(1 - \text{coſ.}\,\psi)}}{m^2 + \mu^2 - 2m\mu\,\text{coſ.}\,\psi}.$$

The angle x defines therefore the poſition of the axis and the two anomalies required, the perihelial diſtance being $p = 2\mu \pm 2\mu \text{ ſin. } x$, it will be known alſo by the angle x.

In order to find the time the comet employs in running its anomalies, let the perihelial diſtance juſt now inveſtigated p be equal to the radius of the earth's orbit, the parabolic area ſwept by the radius vector will be by the nature of the parabola $\frac{2}{3}\text{PO} \times \text{OM} + \frac{1}{2}\text{SO} \times \text{OM} = \frac{4\text{PO} \times \text{OM} + 3\text{SO} \times \text{OM}}{6}$. Now the periphery of the earth's orbit is $7 : 22 :: 2p : \frac{44}{7}p$; therefore the whole area $\frac{44}{7}p \cdot \frac{1}{2}p = \frac{22}{7}p^2$. It is known that the velocity of a heavenly body moved in a circular path, is to the velocity in a parabolic path in the ratio $\sqrt{2} : 1$. If the parabolic area of the comet is divided by $\sqrt{2}$ it comes out $\frac{4\text{PO} \times \text{OM} + 3\text{SO} \times \text{MO}}{6\sqrt{2}}$ equal to an area that the earth deſcribes in the very ſame time; put therefore A equal to the time of a ſidereal year, we ſhall recover the analogy; the whole area of the earth's orbit is to the time in which it is deſcribed as the parabolic area is to the time conſumed in ſweeping it; therefore $\frac{22}{7}p^2 : \text{A} :: \frac{(4\text{PO} + 3\text{SO})\,\text{MO}}{6\sqrt{2}} : \frac{7\text{A}\,(4\text{PO} + 3\text{SO})\,\text{MO}}{72p^2\sqrt{2}}$; but $\text{OM} = \text{SM} \,.\, \text{ſin. anom. PSM}$ and $\text{OS} = \text{SM} \,.\, \text{coſ. anom. PSM}$; let the anomaly be $= \delta$, we have $\text{OM} = m \text{ ſin. } \delta$, and $\text{OS} = m \text{ coſ. } \delta$; therefore $\text{PO} = p - m \text{ coſ. } \delta$. Subſtituting we obtain $\frac{7\text{A}\,(4p - 4m \text{ coſ. } \delta + 3m \text{ coſ. } \delta)\, m \text{ ſin. } \delta}{72p^2\sqrt{2}}$ which is $\frac{7\text{A}\,(4p - m \text{ coſ. } \delta)\, m \text{ ſin. } \delta}{72p^2\sqrt{2}}$, whereby the time is found in parts of a ſidereal year.

I am, &c.

SIR,

SIR, Lyons, May 4, 1783.

LATELY I received from the Obſervatory at Marſeilles the obſervation of the tranſit of Mercury, which happened the 12th Nov. 1782. The ſky not being very favourable, only the two internal contacts were obſerved; the firſt internal contact was obſerved by M. ST. JACQUES DE SYLVABELLE, at 3 h. 18′ 30″ apparent time; the laſt internal contact by M. ST. JACQUES, at 4 h. 30′ 16″; by M. BERNARD, his Adjunctus, at 4 h. 29′ 13″. The neareſt diſtances of Mercury's limb to that of the ſun in the northern part of its diſk were at

h.	′	″	
3	33	14	31
3	42	57	34
4	22	17	19

parts of the micrometer.

The apparent diameter of the ſun was 2174 parts of this micrometer: I ſuppoſe the before-mentioned 2174 parts = 32′ 26″,9. I conclude farther, by the obſervations, the middle of the tranſit at 3 h. 54′ 7″,25, whereas I fix, by interpolation, the diſtances of the limbs at 3 h. 54′ 7″,25 = 35″,6; I have therefore ſemi-diameter of the ſun = 16′ 13″,4 − 35″,6 = 15′ 37″,8 + ſemi-diameter of Mercury = 6″ = 15′ 43″,8 = to the leaſt diſtance of centers of the ſun and Mercury. By M. DE LA LANDE's tables it is 15′ 42″, only a difference of 1″,8.

M. WALLOT at Paris has obſerved this tranſit at the Royal Obſervatory,

	h.	′	″
Firſt external contact	2	56	28
Firſt internal contact	3	2	3
Second - -	4	17	18
Second external -	4	22	53

I only add an important remark upon the diameter of Mercury, which the aſtronomers ſuppoſed in this tranſit = 12″.

Let ABC repreſent the ſun's diſk; in P an external in Q an internal contact; ANC the apparent path of Mercury over the ſun.

The ſemi-diameter of the ſun = 972″, this of Mercury in our ſuppoſition = 6″, MN = 942″ the leaſt diſtances of the centers. In the right-angled triangle MNP it is MP = 972″ + 6″ = 978″, MQ = 972″ − 6″ = 966″; therefore NP will be found = 260″ and NQ = 210″: now NP − NQ = PQ = 50″, which converted into time gives 8′ 14″ for the time the diameter of Mercury employed to run over the limb of the ſun; but by the obſervations of M. Wallot I find this time conſtantly in both contacts 5′ 35″; therefore 8′ 14″ : 12″ :: 5′ 35″ : 8″,137, which ſhould be the diameter of Mercury; and indeed M. Wallot, by an immediate meaſure, taken with an excellent wire-micrometer, finds this apparent diameter not greater than 9″, which ſufficiently ſhews that this diameter ſuppoſed 7″ in the mean diſtance is alſo too great.

I am, &c.

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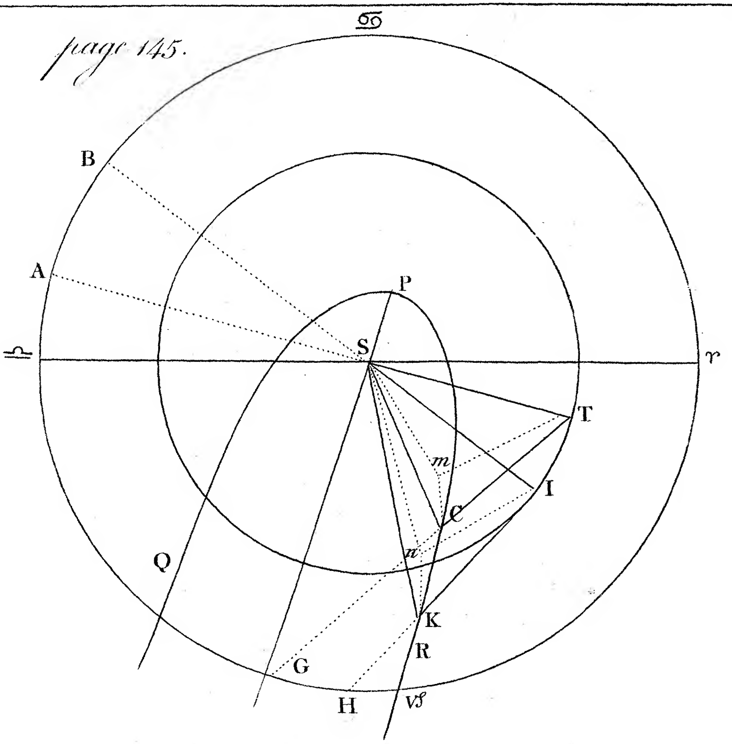

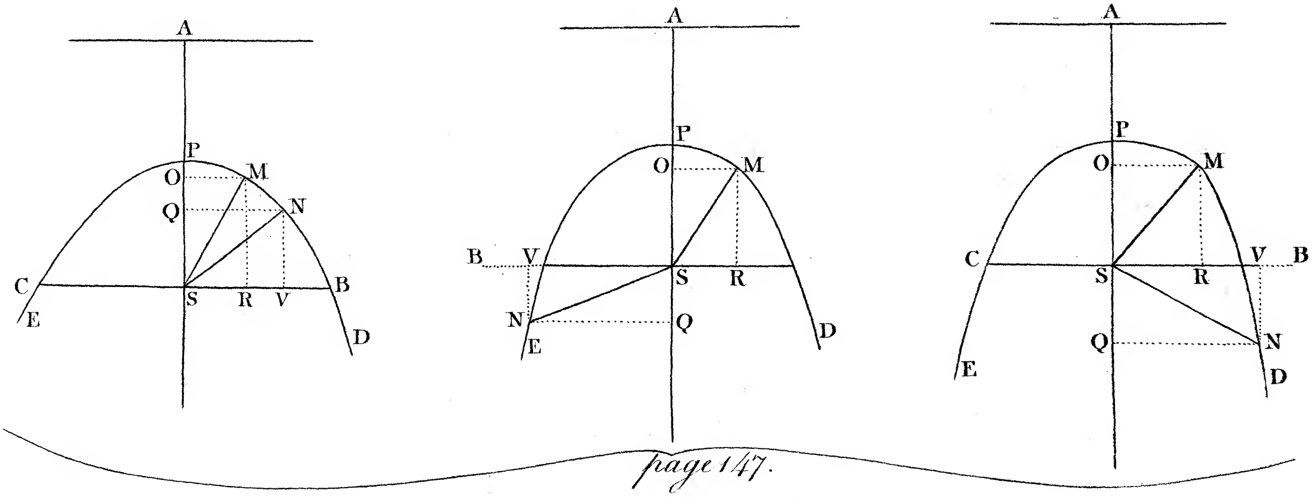

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